

Things to Remember

When n is a positive integer :

01. $(a + x)^n = a^n + {}^n C_1 a^{n-1} x + {}^n C_2 a^{n-2} x^2 + \dots + {}^n C_r a^{n-r} x^r + \dots + x^n$ $[t_r = {}^n C_r a^{n-r} x^r]$

02. $(1 + x)^n = 1 + {}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_r x^r + \dots + x^n$ $[t_r = {}^n C_r x^r]$

03. $(a - x)^n = a^n - {}^n C_1 a^{n-1} x + {}^n C_2 a^{n-2} x^2 - \dots + (-1)^n x^n$

04. $(1 - x)^n = 1 - {}^n C_1 x + {}^n C_2 x^2 - \dots + (-1)^n x^n$

05. Middle Term of $(1 + x)^n$:

(a) when n is even M.T. = ${}^n C_{n/2} x^{n/2}$ (b) when n is odd M.T. = ${}^n C_{\frac{n-1}{2}} x^{\frac{n-1}{2}}$ and ${}^n C_{\frac{n+1}{2}} x^{\frac{n+1}{2}}$

06. Greatest Co-efficient of $(a + x)^n$ or $(1 + x)^n$:

(a) when n is even G.C. = $\left(\frac{n}{2} + 1\right)$ th i.e., middle term's co-efficient.

(b) when n is odd G.C. = $\left(\frac{n-1}{2} + 1\right)$ th or $\left(\frac{n+1}{2} + 1\right)$ th term has the greatest co-efficient, both being equal.

07. Greatest Term of $(a + x)^n$ when $x, a > 0$.

$$\frac{t_{r+1}}{t_r} = \frac{n-r+1}{r} \cdot \frac{x}{a}$$

$t_{r+1} > = < t_r$ according as $r < = > \frac{n+1}{x+a} x$.

08. Properties : (a) $C_0 + C_1 + C_2 + \dots + C_n = 2^n$. (b) $C_1 + C_2 + C_3 + \dots + C_n = 2^n - 1$.

(c) $C_0 + C_2 + C_4 + \dots = C_1 + C_3 + C_5 + \dots = 2^{n-1}$.

Infinite Binomial

09. Expansion for any value of n ($n \neq$ a positive integer)

(a) if $a^2 > x^2$ then $(a + x)^n = a^n + {}^n C_1 a^{n-1} x + {}^n C_2 a^{n-2} x^2 + \dots \infty$

(b) if $a^2 < x^2$ then $(a + x)^n = x^n + {}^n C_1 x^{n-1} a + {}^n C_2 x^{n-2} a^2 + \dots \infty$

10. Deduction : $n \neq$ a positive integer [$-1 < x < 1$ or $|x| < 1$]

(a) $(1 + x)^n = 1 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots \infty$ (b) $(1 - x)^n = 1 - {}^n C_1 x + {}^n C_2 x^2 - {}^n C_3 x^3 + \dots \infty$

For : $-n \neq$ a positive integer

(c) $(1 - x)^{-n} = 1 - {}^n C_1 x + {}^n C_2 x^2 - {}^n C_3 x^3 + \dots \infty$ (d) $(1 - x)^{-n} = 1 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots \infty$

11. Important Results : when $-1 < x < 1$

(a) $(1 + x)^{-1} = 1 - x + x^2 - x^3 + \dots \infty$

(d) $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots \infty$

(b) $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \dots \infty$

(e) $(1 + x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \dots \infty$

(c) $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots \infty$

(f) $(1 - x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \dots \infty$

(g) $(1 - x^2)^{-\frac{1}{2}} = 1 + \frac{1}{2}x^2 + \frac{1.3}{2.4}x^4 + \frac{1.3.5}{2.4.6}x^6 + \dots \infty$

Sum - HS

01. Obtain the term free from x in the expansion $\left(2x + \frac{1}{3x^2}\right)^9$ H-88,78,Taki,SCH-88,Joy-89,Rah-86

02. Find the co-efficient of x in $(1 - 2x^3 + 3x^5)\left(1 + \frac{1}{x}\right)^{10}$ H-79,Ashu-83,Jaga-84,PBh-85,Taki,Xav-86,PBh-88

03. If $-1 < x < 1$, prove that H-02,85,79,Xav,SPT-87,Jaga-84,Ashu-83

$(1 + x + x^2 + x^3 + \dots \infty)(1 + 2x + 3x^2 + 4x^3 + \dots \infty) = \frac{1}{2}(1.2 + 2.3x + 3.4x^2 + \dots \infty)$

04. Find the co-efficient of x^r in the expansion $\frac{1+x}{(1-x)^2}$ H-80,SCh-83

05. Find the co-efficient of x^{16} in the expansion $x^{10}(x-2)^{10}$. H-81,SCh-86,Taki-87

06. Given that $(1 - x)^n = C_0 - C_1x + C_2x^2 - C_3x^3 + \dots + (-1)^n C_r x^r + \dots + (-1)^n x^n$; show that $C_1 + 2C_2 + 3C_3 + \dots + nC_n = n \cdot 2^{n-1}$. **J-82, H-83,81,Viv,Rah-88,Jaga,SCh-86,Beth-85,Hare-83**
07. Find the first three terms in the expansion of $(2 + 3x)^{-3}$ and state the condition of validity of the expansion. **H-81,Belur-88,87,Rah-86**
08. Write down the first four terms in the infinite series expansion of $(1 + x)^{1/4}$ in ascending powers of x and state the condition of validity of the expansion. **H-82,Xav-87,Viv-88**
09. Find the term independent of x in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ **J-84, H-90,82,City-87,SPT-88,Nar,SCh-86,Ashu-83**
10. Prove that the co-efficient of the middle term in the expansion of $(1 + x)^{12}$ is equal to the sum of the co-efficients of the two middle terms in the expansion of $(1 + x)^{11}$. **H-84,82**
11. Find the middle term in the expansion of $\left(\frac{x^2}{3} + \frac{3}{x^3}\right)^8$ **H-83,Rah-87,Xav-86,85,83,Ashu-85,Joy-83**
12. If in the expansion of $(1 + x)^{32}$, the co-efficient of the $(3r + 1)$ th term be equal to the co-efficient of the $(r + 5)$ th term, show that the value of r is 7. **J-75,H-83, Uttar,Jaga-86,AV-83**
13. If $|x| < \frac{1}{2}$, find the co-efficient of x^n in the expansion of $(1 - 2x)^{-\frac{1}{2}}$ **H-83**
14. Which term in the expansion of $(1 + x)^p \left(1 + \frac{1}{x}\right)^q$ is independent of x , where p and q are positive integers ?
What is the value of the term ? **H-89,84,Bally,Mitra-88,Hindu-86, Hare-85**
15. If n is a positive integer and $|x| < 1$, show that the co-efficient of the n th term in the expansion of $(1 - x)^{-n}$ is twice the co-efficient of the $(n - 1)$ th term. **H-84,Xav-85**
16. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, prove that $C_0 + 2C_1 + 3C_2 + \dots + (n + 1)C_n = (n + 2)2^{n-1}$. **H-85,Joy-89,City-88,Rah-86,SCh-85**
17. Find the co-efficient of x^{-2} in the expansion of $\left(3x - \frac{7}{x}\right)^8$. **H-86,Viv-86,Nar,Bally-83**
18. Write down the first four terms in the expansion of $(2 + x)^{-2}$ in ascending power of $\frac{1}{x}$, stating the condition of validity of the expansion. **H-86**
19. Use binomial theorem to show that if $n \geq 2$ is an integer, then $11^n - 10n - 1$ is **divisible** by 100. **H-86, PBh-88**
- 19a. Use binomial theorem to show that if $n \geq 2$ is an integer, then $3^{2n+2} - 8n - 9$ is **divisible** by 64. **IIT-77, HS-96**
20. Use binomial theorem to calculate $(1.02)^8$ correct to two places of decimals. **HS-87, Ashu-85**
21. Examine whether there is any term containing x^9 in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{20}$. **HS-87**
22. If $y = 2x + 3x^2 + 4x^3 + \dots \infty$ for $|x| < 1, |y| < 1$, find the first three terms in the expansion of x in a series of ascending powers of y . **HS-87, AV-83**
23. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, show that $C_0C_n + C_1C_{n-1} + C_2C_{n-2} + \dots + C_nC_0 = \frac{|2n}{(\underline{n})^2}$ **H-88**
24. The condition of validity of the expansion being assumed to be satisfied, find the fifth term in the expansion of $(1 - 2x)^{-\frac{1}{2}}$ in ascending powers of x . [give your answer in the simplified form]. **J-84,H-88,Nar-85**
25. Express the series $2 + \frac{5}{\underline{2.3}} + \frac{5.7}{\underline{3.3^2}} + \frac{5.7.9}{\underline{4.3^3}} + \dots$ to ∞ in the form of a binomial expansion and hence find its value. **J-80, SCh-85**
26. Find the **numerically greatest term** (or terms) in the expansion of $(2 - 3x)^{13}$ when $x = \frac{1}{2}$ **PBh,Belur,Uttar-88 PBh-87, SCh-86**
27. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, prove that $C_0 + 3C_1 + 5C_2 + \dots + (2n + 1)C_n = (n + 1)2^n$. **Taki-87,85, Viv-83**
28. If $|a| < 1$ and $|b| < 1$, then find the sum of the series : $1 + (1 + a)b + (1 + a + a^2)b^2 + (1 + a + a^2 + a^3)b^3 + \dots$
29. The successive co-efficients in the expansion of $(1 + x)^n$ is a, b, c , show that $n = \frac{2ac + b(a + c)}{b^2 - ac}$.



Sum - HP

01. If in the expansion of $(1 + ax + 2x^2)^6$ the co-efficients of x^2 and x^{11} be 27 and -192 respectively, then prove that $a = -1$. J-66

02. Sum to infinity : $1 + \frac{1}{2} \cdot \left(\frac{2}{3}\right) + \frac{1}{2} \cdot \frac{3}{4} \left(\frac{2}{3}\right)^2 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \left(\frac{2}{3}\right)^3 + \dots \infty$ J-66, Joy-86

03. Sum to infinity : $\frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \left(\frac{1}{4}\right) + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \left(\frac{1}{4}\right)^2 + \dots \infty$ J-68, Taki-88, Nar-87

04. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, show that $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$.
H-98,95, J-82,69, I-75, SPt-06, Bally, Kanai-88, Hind, PBh-87, Joy-86, Hind, City-85, Nar, SCh-83

04(a). If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, show that $\frac{C_0}{1} - \frac{C_1}{2} + \frac{C_2}{3} - \dots + (-1)^n \frac{C_n}{n+1} = \frac{1}{n+1}$. SPt-01

05. If P_n be the product of all the co-efficients in the expansion of $(1 + x)^n$, then prove that $\frac{P_{n+1}}{P_n} = \frac{(n+1)^n}{n}$.
J-70, Mitra-88, Nar-86, PBh-85, Hooghly-84, Taki-83

06. In the expansion of $(1 + x)^n$, if the numerical co-efficients of any four consecutive terms be a_1, a_2, a_3, a_4 , then prove that $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} + \frac{2a_2}{a_2 + a_3}$. J-71, I-75

07. If c be such a number that c^4 can be neglected in comparison with l^4 , then show that

$$\sqrt{\frac{l}{l+c}} + \sqrt{\frac{l}{l-c}} = 2 + \frac{3}{4} \cdot \frac{c^2}{l^2}$$
J-74, PBh-87,83

08. Find the index of the power of x in the greatest co-efficient in the expansion of $\left(2 + \frac{5x}{2}\right)^{12}$ J-76

09. If $y = 3x + 6x^2 + 10x^3 + \dots \infty$ then prove that $x = \frac{y}{3} - \frac{1 \cdot 4}{3^2 \cdot \sqrt{2}} y^2 + \frac{1 \cdot 4 \cdot 7}{3^3 \cdot \sqrt{3}} y^3 - \dots$ to ∞ J-85

10. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, show that $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3) \dots (C_{n-1} + C_n) = \frac{(n+1)^n}{n} C_1 C_2 C_3 \dots C_n$ J-86, Nar-88,87, Hind-84

11. Sum to infinity : $\frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} + \dots \infty$ [1] Nar-86

12. If $x = 1 + a + a^2 + \dots$ to ∞ and $y = 1 + b + b^2 + \dots$ to ∞ , where a and b are proper fractions, then the value of $z = 1 + ab + a^2 b^2 + \dots$ to ∞ is given by (i) $\frac{xy}{x+y+1}$ (ii) $\frac{xy}{x+y-1}$
 (iii) $\frac{xy}{x-y+1}$ (iv) $\frac{xy}{x-y-1}$. Bri, Hindu-87,83

13. If n be a positive integer, and if the third, fourth, fifth and sixth terms in the expansion of $(x + p)^n$ be a, b, c, d respectively, prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{5a}{3c}$. Hindu-88,86

14. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, show that $C_1^2 + 2C_2^2 + 3C_3^2 + \dots + nC_n^2 = \frac{\sqrt{2n-1}}{(\sqrt{n-1})^2}$ J-90

15. Sum to infinity : $1 + \frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 8} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 8 \cdot 12} + \dots \infty$ [$\sqrt{2}$] J-07, H-89,69,67, Mitra-88, City-86, SCh-85,83

16. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, prove that $C_1 - 2C_2 + 3C_3 - \dots + (-1)^{n-1} n \cdot C_n = 0$. H-72, Moulana-88, SCh-86

17. If $y = x + x^2 + 2x^3 + \dots + \frac{\sqrt{2n}}{n} x^{n+1} + \dots$, prove that $y^2 - y + x = 0$

18. Show that the middle term of the expansion $(1 - x)^{2n}$ is $\frac{(-1)^n \cdot 1 \cdot 3 \cdot 5 \cdot 7 \dots (2n-1) \cdot 2^n \cdot x^n}{n}$ H-89, Vidyasagar-85

19. Sum to infinity : $1 + \frac{1}{6} + \frac{1 \cdot 4}{6 \cdot 12} + \frac{1 \cdot 4 \cdot 7}{6 \cdot 12 \cdot 18} + \dots \infty$ [$\sqrt[3]{2}$] SPt-87,84, Xav, Jaga-86

20. Sum to infinity : $1 + \frac{10}{9} + \frac{10 \cdot 16}{9 \cdot 18} + \frac{10 \cdot 16 \cdot 22}{9 \cdot 18 \cdot 27} + \dots \infty$ [$3\sqrt[3]{9}$] Hind-87, Rah-86
21. Sum to infinity : $1 + \frac{4}{6} + \frac{4 \cdot 5}{6 \cdot 9} + \frac{4 \cdot 5 \cdot 6}{6 \cdot 9 \cdot 12} + \dots \infty$ [$\frac{19}{8}$] Narendranath-83
22. Show that $\sqrt{2} = \frac{7}{5} \left\{ 1 + \frac{1}{10^2} + \frac{1 \cdot 3}{1 \cdot 2} \cdot \frac{1}{10^4} + \frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \cdot \frac{1}{10^6} + \dots \infty \right\}$ Hind-83
23. If $y = x - x^2 + x^3 - x^4 + \dots \infty$ when $-1 < x < 1$, prove that $x = \frac{y}{1-y}$ Bally-83
24. Find the co-efficient of x^n in the expansion of $\frac{x}{(1-2x)(1-3x)}$. SPt-86, StL-88
25. If 3 consecutive co-efficients in the expansion of $(1+x)^n$ be 66, 220, 495, find the value of n. [12] PBh-87, StL-88
26. Show that $\sqrt{\frac{1+x}{1-x}} = 1 + \left(\frac{x}{1+x}\right) + \frac{3}{2} \cdot \left(\frac{x}{1+x}\right)^2 + \frac{5}{2} \cdot \left(\frac{x}{1+x}\right)^3 + \dots \infty$ Hind-88,86
27. If x is so small that x^2 and higher power of x may be neglected, show that $\frac{\sqrt{1+x} + \sqrt[3]{(1+x)^2}}{1+x + \sqrt{1+x}} = 1 - \frac{x}{6}$. J-73
28. Show that $(x-1)^2$ is a factor of $x^n - nx + n - 1$.
29. Find the term independent of x in the expansion of $(1+x)^m \left(1 - \frac{1}{x}\right)^n$, where m and n are positive integers ?

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Sum - TP

01. If $C_0, C_1, C_2, C_3, \dots, C_{15}$ are the binomial co-efficients in the expansion of $(1+x)^{15}$, prove that $\frac{C_1}{C_0} + 2 \cdot \frac{C_2}{C_1} + 3 \cdot \frac{C_3}{C_2} + \dots + 15 \cdot \frac{C_{15}}{C_{14}} = 120$. SPt-02, I-61, Xav-85, Uttar-88
02. Evaluate $(0.99)^{15}$ correct to four places of decimals. I-64, Xav-88 [T-02]
03. Find the **term independent** of x in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{\sqrt{3}}{2x^2}\right)^{10}$ I-65, Xav-88 [T-04]
04. If the co-efficients of $(2r+4)$ th, $(r-2)$ th terms in the expansion of $(1+x)^{18}$ are equal, find r. I-64, Law-88 [T-05]
05. Find the value of **greatest term** in the expansion of $\sqrt{3} \left(1 + \frac{1}{\sqrt{3}}\right)^{20}$ I-65, PBh-88 [T-06]
06. If $(1+x)^{15} = C_0 + C_1x + C_2x^2 + \dots + C_{15}x^{15}$, find the value of $C_2 + 2C_3 + 3C_4 + \dots + 14C_{15}$ [212993] I-66 [T-07]
07. If the co-efficients of 2nd, 3rd and 4th terms in the expansion of $(1+x)^{2n}$ are in A.P. show that $2n^2 - 9n + 7 = 0$. J-07, I-68, Law-87 [T-09]
08. Find the **term independent** of x in the expansion of $(1+x+x^2) \left(\frac{3}{2}x^2 + \frac{1}{3x}\right)^9$ I-62, Hind-87 [T-12]
09. If $C_0, C_1, C_2, C_3, \dots, C_n$ be the co-efficients in the expansion of $(1+x)^n$, prove that
- (a) $C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2 = \frac{2n}{(\ln)^2}$ H-00,97, I-73, Vidya-85 [T-14]
- (b) $C_0C_r + C_1C_{r+1} + C_2C_{r+2} + \dots + C_nC_{n-r} = \frac{2n}{\frac{n+r}{n} \cdot \frac{n-r}{n}}$ I-74, Belur-87,86, Hind-83 [T-14]
10. Show that the larger of $(99)^{50} + (100)^{50}$ and $(101)^{50}$ is $(101)^{50}$ I-82 [T-19]
11. Show that the sum of the co-efficients of the polynomial $(1+x-3x^2)^{2143}$ is (-1) . I-82 [T-19]
12. Find the **term independent** of x in $(1+x)^2 \left(x - \frac{1}{x}\right)^7$ [70] [K-41]
13. Show that the **middle term** in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is $\frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n!} (-2)^n$. [K-42]

Giving xerox of these sums is strictly prohibited